

Homework from Friday: you can do it all without a calculator!

Go slowly through this solution and ask questions of each other as needed. This sheet is available on our class web site if you need it.

$$s(t) = (2/5)t^5 - (8/3)t^3 \\ = t^3(0.4t^2 - 2.333) \text{ (Notice there are 3 roots with odd multiplicity).}$$

This tells us that the starting position ($t = 0$) is 0. Notice end behavior: negative infinity on the left and positive infinity on the right. Visualize the graph: comes up from negative infinity, crosses the x-axis somewhere $t < 0$, crosses again at 0, crosses again with $t > 0$ and continues up toward infinity.

$$v(t) = 2t^4 - 8t^2 \\ = 2t^2(t^2 - 4) \\ = 2t^2(t - 2)(t + 2)$$

This tells us that the starting velocity is 0 (even multiplicity: particle stops momentarily before continuing in the same direction), and velocity is zero at 2 and -2 (odd multiplicity: particle changes directions). Notice end behavior: positive infinity in both directions. Visualize the graph: coming down from infinity, crossing x-axis at -2 , coming back up, touching x-axis at 0, going back down, coming back up to cross x-axis at 2 and continue up to infinity.

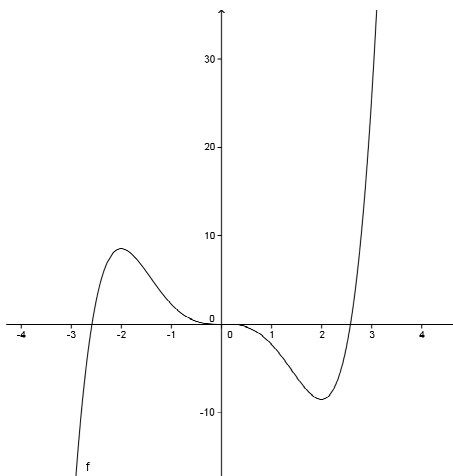
$$a(t) = 8t^3 - 16t \\ = 8t(t^2 - 2)$$

This tells us that the starting acceleration is 0, and acceleration is also zero at $t = \pm\sqrt{2}$.

So the particle is moving right (at $t < -2$), changes direction at time $= -2$, slows to a momentary stop at time $= 0$, continues moving left until time $= 2$ when it changes direction again, continuing right at increasing velocity forever. Relative maximum speeds occur at $t = \pm\sqrt{2}$ (where velocity $= -8$). Particle is picking up speed (when velocity > 0 and increasing or $v < 0$ and decreasing) on $t = (-2, -\sqrt{2})$ and $t = (0, \sqrt{2})$ and $t = (2, \text{infinity})$; particle is losing speed at other times (except when it has paused momentarily, at $-2, 0, 2$).

If you are allowed a calculator, you would graph the functions to confirm your ideas:

Position



Velocity

